QUESTION 1

Given:

$$f(x) = -2\sin(3\pi x) + 4$$

Let:

$$\begin{aligned} A &= \text{the amplitude of } f(x) \\ B &= \text{the period of } f(x) \\ C &= \text{the maximum value of } f(x) \\ D &= \text{the minimum value of } f(x) \\ E &= f\left(\frac{1}{4}\right) \end{aligned}$$

Find $ABCDE$.

QUESTION 2

Let:

$$A = < 1, 1, 2 > \bullet < 0, 1, 8 >$$
$$B = ||< 1, 1, 2 > \times < 0, 1, 8 >||$$
$$C = \text{the positive difference of the eigenvalues of the matrix } \begin{bmatrix} 3 & -1 \\ 2 & 6 \end{bmatrix}$$
$$D = \text{the volume of a parallelepiped determined by the vectors } < 1, 0, 8 >, < 9, 3, -2 >, < 1, 2, -2 >$$

Find $A + B^2C + D$.

QUESTION 3

Let:

$$A = \sin(2x), \text{ if } \sin(x) + \cos(x) = \frac{5}{4}$$

$$B = \text{the number of solutions to } \sin 18x = 0, \text{ where } 0 \leq x \leq 2\pi$$

$$C = \tan\left(\arccos\left(\frac{20}{29}\right)\right)$$

$$D = \frac{\sin^2(10^\circ) + \sin^2(20^\circ) \ldots + \sin^2(80^\circ) + \sin^2(90^\circ)}{\cos^2(10^\circ) + \cos^2(20^\circ) \ldots + \cos^2(80^\circ) + \cos^2(90^\circ)}$$

Find $(B-1)D + 80(A+C)$.

QUESTION 4

Tanvi, the small dragon, loves to fly! Starting from her castle, Tanvi flies at a remarkable speed of 4 m/s 45° north of east. The wind flows at a speed of 6 m/s 30° north of east. Let $A =$ the distance, in meters, between the castle and Tanvi after the latter has flown for 3 seconds.

Points $P_1, P_2, P_3, \ldots, P_{24}$ are equally spaced (in this order) around the circumference of a circle C which has a radius of 2. The line segment connecting P_2 and P_{24} intersects the line segment connecting P_1 and C at point X. Let $B =$ the sum of the squares of the distances from X to each of $P_1, P_2, P_3, \ldots, P_{24}$.

Find $A + B$.

QUESTION 5

Given:

$$A = \begin{bmatrix} 3 & 0 & 4 \\ 1 & -7 & 1 \\ 4 & 4 & 3 \end{bmatrix} \quad B = \begin{bmatrix} 4 & 3 & -4 \\ 0 & 7 & 1 \\ -1 & 1 & 3 \end{bmatrix}$$

Let:

$$\begin{aligned} V &= \text{the trace of } A(B^T) \\ W &= \text{the determinant of } B^{-1} \\ X &= \text{the determinant of } -\frac{A^2}{106} \\ Y &= |C| \text{ if } C \text{ is a } 3 \times 3 \text{ matrix such that } AC = \begin{bmatrix} 15 & 10 & 3 \\ 11 & -11 & -13 \\ 9 & 19 & 12 \end{bmatrix} \end{aligned}$$

Find $V + 343W + 4X + 12Y$.

QUESTION 6

Let $A =$ the area of a triangle with vertices $(1, 4, 2)$, $(4, 8, 3)$, and $(7, 12, 3)$.

Sanjita has always been afraid of heights, but she has finally decided to face her fear by going on a Ferris Wheel! The Ferris Wheel has a height of 200 ft and a radius of 40 ft. Sanjita's car is currently 180 feet above the ground (she's starting to cry!) and is moving at 30 ft/sec. Let $B =$ Sanjita's angular velocity in radians per second.

Deekshita enjoys twirling her eraser when she is bored. Her eraser is in the shape of a trapezoid, $ABCD$ in which $\overline{AB}$ is parallel to $\overline{CD}$. $\overline{AB} = 5$, $\overline{BC} = \sqrt{13}$, $\overline{CD} = 8$, and $\overline{AD} = \sqrt{10}$. She positions the eraser such that $\overline{CD}$ lies on a flat horizontal table and $\overline{AB}$ is directly above $\overline{CD}$. She continuously rotates the eraser around D such that $\overline{CD}$ is always in full contact with the tables and $\overline{AB}$ is at a constant height above $\overline{CD}$. Let $C =$ the volume of the solid of revolution formed by this rotation (assume the eraser's thickness is negligible).

Find $AB + C$.

QUESTION 7

Let:

$$A = \text{the eccentricity of the polar curve } r = \frac{16}{2 - 12\cos\theta}$$

$$B = \text{the value such that the area of the conic } 7x^2 + Bxy + y^2 = 1 \text{ is } \frac{\pi}{2}$$

Given:

$$f(x) = y^2 - 32x - 4y + 4 = 0$$

Let:

$$C = \text{the abscissa of the endpoint of the latus rectum } f(x) \text{ in Quadrant I.}$$

$$D = \text{the ordinate of the endpoint of the latus rectum } f(x) \text{ in Quadrant I.}$$

$$E\pi = \text{the area of the region enclosed by the graph of } r = 12\sin\theta$$

Find $\dfrac{AD}{E} + BC$.

QUESTION 8

Let:

$$A = \lim_{x \to 4} \frac{2 - \sqrt{x}}{4 - x}$$
$$B = \lim_{x \to \infty} \frac{1}{x}$$
$$C = \lim_{x \to 3} \frac{x^3 - x^2 - 41x + 105}{x^2 - x - 6}$$
$$D = \lim_{x \to \infty} \sqrt{9x^2 - 12x} - 3x$$

Find $AB + CD$.

QUESTION 9

Let:

$$A = (\cos(35^\circ) + i\sin(35^\circ))^6$$
$$B = (\cos(60^\circ) + i\sin(60^\circ))^6$$
$$C = (\cos(21^\circ) + i\cos(69^\circ))^{15}$$
$$D = 5\operatorname{cis}45^\circ$$

If $2(A + C + D) + B = 1 + a\sqrt{3} + b\sqrt{2} + ci + d\sqrt{2}i$
Find $a + b + c + d$.

QUESTION 10

Let:

$A =$ the number of petals in the polar curve $r = 360\sin(45\theta)$

$B =$ the length of a petal, from the origin to the tip, in the polar curve $r = 4\sin(2\theta)$

$C =$ the number of intersections between $r = 4$ and $r = 8\cos(6\theta)$

$D =$ the eccentricity of the polar curve, $r = \dfrac{3}{-4 + 2\cos(\theta)}$

Find $BCD + A$.

QUESTION 11

Rayyan, Anirudh, Siddharth, Nihar, Josh, and Deekshita participate in a tic-tac-toe tournament. This is a round-robin tournament (each player plays one game with every other player) and the probability that any player wins a given game against anyone else is $\frac{1}{2}$, with the exception that Rayyan will always defeat Anirudh and Nihar will always defeat Josh. Let $A =$ the probability that there is no player who ends up with 0 wins or 0 losses.

Consider a conic section defined by the equation $7x^2 - 6\sqrt{3}xy + 13y^2 - 16 = 0$. If this conic section is rotated in a $x'y'$-system such that the $x'y'$ term is eliminated, let $B =$ the sum of the coefficients of the equation that defines the rotated conic section. (Note: the equation of the rotated conic section should be in the form $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$ where the greatest common factor of all the coefficients is 1.)

Find $A + B$.

QUESTION 12

Consider an ellipse defined by the equation $\frac{(x-4)^2}{25}+\frac{(y-2)^2}{16}=1$. A triangle is circumscribed about the ellipse. Two sides of the triangle are tangent to the ellipse at the endpoints of the ellipse's latus recta with the highest ordinate value. The third side of the triangle is tangent to the ellipse at the co-vertex with the lowest ordinate value. Let $A=$ the area of the triangle.

$$\begin{aligned} B &= \lim_{x\to\infty}\frac{\sin(7x)}{x^2\csc(3x)} \\ C &= \text{the period of } f(x)=\cos(\cos(\cos(x)))\sin(3x) \\ D &= \text{the value of } z \text{ when } \sum_{n=0}^{3}\sum_{a=1}^{2}\sum_{x=0}^{\infty}\frac{an^x}{x!}=ye^0+ye^1+\ldots+ye^z \text{ where } y \text{ and } z \text{ are integers} \\ &\ \text{with definite values and } e \text{ is the mathematical constant (Euler's number)} \end{aligned}$$

Find $\frac{A+B+C}{\pi+D}$.

QUESTION 13

Let:

$$A = \text{the number of integer solutions of } 2^{x+1} + 4^x = 3$$

Given:

$$f(x) = 2x - 15x^{\frac{2}{3}} + 28x^{\frac{1}{3}}$$

Let:

$B =$ the smallest solution to $f(x)$
$C =$ the greatest solution to $f(x)$
$D =$ the 12^{th} term of a harmonic sequence, given that the 4^{th} term is $\dfrac{4}{5}$ and 7^{th} term is $\dfrac{1}{2}$
$E =$ the distance between the lines $4x + 3y = 33$ and $4x + 3y = 1$

Find $A + B + C + D + E$.

QUESTION 14

$$\begin{vmatrix} 3 & 1 & 0 & 7 \\ 4 & -5 & 0 & -1 \\ 2 & 5 & 2 & 2 \\ 0 & 2 & -3 & 6 \end{vmatrix}$$